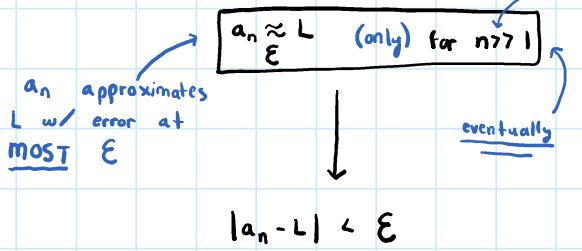
Limit

Wednesday, April 5, 2023 9:31 AM

def: given a sequence, $L \in$ a real # ($\mathbb{R}$), a_n is said to have limit (L) (denoted $\lim_{n\to\infty} a_n = L$, $a_n \to L$) for any small error (ε) we want ...



* ex) $\varepsilon = 1$ $L = \pi$

$a_n = 4, 3$

ex) $\varepsilon = 0.1$ $L = \pi$

$a_n = 3.2, 3.1$ *

hierarchy of growth (test convergence):

$$1 \ll \ln(n) \ll n^{\alpha} \ll a^n \ll n! \ll n^n$$

constants; polynomials (ex: $\sqrt{n}$); expanentials

- means when $n \to \infty$, f is faster than g if $g \ll f$ (when $n \to 0$) ↖ as it gets closer to lim

ex) decide convergence / divergence

1) $\frac{1}{n} \to 0$ converges ✓

2) $a_n = \frac{n}{\ln(n)}$ "$\to$" ∞ diverges

3) $a_n = \frac{\ln(\ln(n))}{2^n} \to 0$ converges ✓

4) $a_n = \frac{n^3 + n + 3}{4n^3 - 2n^2 + 5} \to \frac{1}{4}$ converges ✓ * $\frac{n^3}{n^4} \to 0$ (n^4 grows faster), $\frac{n^4}{n^3} \to \infty$ (n^4 grows faster) *